The Algebra of Shapes

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Abstract

This thesis investigates a new representation scheme for geometric modeling, based on an algebraic model for shapes and formalized using a boundary representation. The algebraic model is mathematically uniform for shapes of all kinds and provides a natural and intuitive framework for mixed-dimensional shapes. The corresponding maximal element representation is essential to the concept of shape emergence. The representation scheme particularly supports computational design as a generative process of search or exploration.

This thesis begins with a treatment of the algebraic and geometric properties of shapes and gives a formal and complete definition of the maximal element representation for n-dimensional shapes in a k-dimensional space ($n \leq k$). Efficient algorithms are presented for the algebraic operations of sum, product, difference and symmetric difference on shapes of plane and volume segments. An exploration of related research in shape grammars, computational design and construction simulation, illustrates the potential of this representation scheme, while an agenda for future research depicts its present shortcomings.